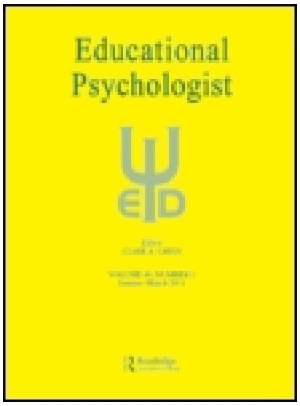

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Rethinking Formalisms in Formal Education

Mitchell J. Nathan [a]
[a] Educational Psychology Department, Department of Psychology, and Department of Curriculum & Instruction, University of Wisconsin, Madison

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Rethinking Formalisms in Formal Education

Mitchell J. Nathan
Educational Psychology Department, Department of Psychology, and Department of Curriculum & Instruction
University of Wisconsin–Madison

I explore a belief about learning and teaching that is commonly held in education and society at large that nonetheless is deeply flawed. The belief asserts that mastery of *formalisms*—specialized representations such as symbolic equations and diagrams with no inherent meaning except that which is established by convention—is prerequisite to applied knowledge. A *formalisms first* (FF) view of learning, rooted in Western dualist philosophy, incorrectly advocates the introduction of formalisms too early in the development of learners' conceptual understanding and can encourage a formalisms-only mind-set toward learning and instruction. I identify the prevalence of FF in curriculum and instruction and outline some of the serious problems engendered by FF approaches. I then turn to promising alternatives that support *progressive formalization*, problem-based learning, and inquiry learning, which capitalize on the strengths of formalisms but avoid some of the most costly problems found in FF approaches.

Well before the systematic study of science and mathematics, people were able to fly kites, solve story problems, and even build bridges and computers. Yet the scholastic experience seems blind to many of these competencies or willfully neglectful of them. Instead, in the hierarchically organized, chronologically graded system typical of formal education, the teaching and mastery of formalisms are often considered prerequisite to applied knowledge. Formalisms occupy this primary role because of deeply held beliefs in education and in society at large in a *formalisms first* (FF) view, which posits that learning and conceptual development proceeds first from knowledge and mastery of discipline-specific formalisms before learners can exhibit competency applying that knowledge to practical and clinical matters.

Formal structures, or *formalisms*, admittedly, constitute a "fuzzy" category, one that does not have clearly specified boundaries. Under what I term the *narrow view of formalisms*, formalisms are confined to specialized representational forms that use heavily regulated notational systems with no inherent meaning except those that are established by convention to convey concepts and relations with a high degree of specificity. The regulated conventions are intended to reduce ambiguity and increase the potential for processing them in systematic and objective ways. Under the terms *formal structure, formal knowledge, formal representations, formal theories, formal models,* and *formalisms* fall symbolic structures and inscriptions (Latour, 1987, 1990; Latour & Woolgar, 1986) from the natural and information sciences such as mathematical expressions and equations, stoichiometric equations, and vector diagrams, Boolean algebra, and computer programming routines; conventional charts and diagrams, such as Cartesian graphs, flow charts, topological graphs; and standardized tabular representations such as data tables, matrices, and balance sheets. Members of this category all have in common that they are conventionalized and transcendent "forms" intended to "ascend" or abstract beyond the particulars of any material manifestation in order to facilitate documentation and be "mobile, but also immutable, presentable, readable, and combinable with one another" (Latour, 1986, p. 7). The narrow view of formalisms is strongly represented in science, technology, engineering, and mathematics (STEM fields). In addition, I consider how a *broad view of formalisms*—which includes scientific theories and models, formal principles, analytic cases, psychological constructs and abstract generalizations—contributes to this portrait of FF in areas such as the social sciences, humanities, and teacher education.

My central claim is that the FF view exerts significant influence on formal education but the view is misguided. Although the case for FF based on the narrow consideration of formalisms in STEM fields garners strong support, I also show that the prevalence of FF in non-STEM fields is also highly suggestive under a broader view of formalisms.

Correspondence should be addressed to Mitchell J. Nathan, Educational Psychology Department, University of Wisconsin–Madison, 1025 West Johnson Street, Madison, WI 53706-1796. E-mail: mnathan@wisc.edu

I argue that FF is inappropriate for two main reasons. First, FF supports an inadequate view of conceptual development. It improperly privileges formal, scientific knowledge over applied knowledge and (as I show) inaccurately advocates the introduction of formalisms as a necessary precursor to understanding and application. Second, FF encourages a formalisms-only mind-set, where formalisms drive out other representations and forms of reasoning. I elaborate on each of these in the coming sections.

Formalisms, it must be acknowledged, are powerful for their abilities to reify tacit knowledge, highlight deep structure, and retain and transmit cultural knowledge. However, care must be taken not to conflate the foundational role formalisms play in specifying the structure of a discipline and in providing conceptual tools for disciplinary experts, with the developmental needs of students who are initially learning the knowledge and practices of the discipline. Despite evidence that FF is a flawed view of learning and development for many students and in many content areas, its influence on instruction, curriculum design, and educational policy is far-reaching. The societal roots of FF are deeply ingrained and operate with little scrutiny.

FORMALISMS AND APPLIED KNOWLEDGE

One of the main ways formal knowledge is privileged is when it is cast as the driving force of scientific advancement and positioned as fundamentally antecedent to applied knowledge. Perhaps the ultimate example of the power of formal, abstract thought to further advance science is the prevailing account of Einstein's (1916) articulation of the theory of relativity. The theory of relativity is the archetypal scientific revolution (Kuhn, 1962; Posner, Strike, Hewson, & Gertzog, 1982). As the account goes, Einstein's process of scientific discovery was both intuitive and abstract, drawing profoundly on the prevailing scientific theories of electromagnetism and inertial frames of reference (Stachel, 1982). Einstein explicated his theory by drawing on isometries within a Poincare Group (drawing on the associated algebraic notation), which provided the formal basis for the Lorentz Transformations that figured prominently in his thinking. Relativity cut to the heart of theoretical physics and the current conceptualization of the nature of the universe. It stood light years apart from the world occupied by tinkerers and manual laborers. Yet, as the circumstances of Einstein's contributions come to light, the account of his discovery is being revised, with a greater appreciation for the role applied knowledge played in Einstein's thinking, calling into question the relationship of theoretical and applied knowledge in physics and throughout the sciences.

Formal Knowledge and Scientific Advancement

The foundation of Einstein's groundbreaking theory was to reframe the question of *simultaneity*—what it meant for two events to happen at the same time even when they were not located at the same place. As it is commonly understood, the solitary theoretician, Albert Einstein, worked in the confines of the Swiss patent office by day, and pondered—indeed, rewrote—Isaac Newton's widely accepted laws of the universe. Einstein's discovery was considered to be exceedingly abstract, couched deeply in theory and inscribed in the language and notation of formal mathematics and physics.

A question that plagued Einstein was how, hypothetically, clocks at different places could accurately mark simultaneous events. In Newton's universe, the solution for synchronizing distant clocks was simple. It drew on the postulate of Absolute Time, where each place could be calibrated to one, common time standard. But young Einstein rejected any method where a central, "master" clock had to send a signal to secondary clocks to denote the simultaneity of an event. Because the secondary clocks could not, in practicality, all be equidistant, and because light travels at a finite speed, clocks closer to the central clock would receive the signal and record the event before the others. To Einstein, it was unacceptable on practical as well as theoretical grounds that the measure of simultaneity of an event should depend on proximity to a central clock. To rectify this shortcoming, Einstein (as cited in Miller, 1981) offered a *gedankenexperiment* (or thought experiment) to investigate legitimate procedures for coordinating time. In his imaginary system, idealized clocks sent electrical signals along idealized cables. Even in this simulated world, Einstein reasoned that if one took into account the time for the signal to travel from a central clock to the different locations of each secondary clock along these cables (with each cable distance divided by the speed of light), then the time recorded by a secondary clock would be independent of its proximity to the master clock. In a further stroke of insight, Einstein showed that there was no need for a central clock at all, because all times could be calibrated relative to their distances (i.e., their *times*) from each other.

In thinking through simultaneity in this newly proposed model of a universe of relative time, Einstein drew on another *gedankenexperiment*, this one addressing the relative experiences of observers on trains headed toward or away from a lightning strike, as compared to a stationary observer positioned along the railway embankment (Einstein, 1916).

The Revisionist Account of Applied Knowledge in Scientific Advancement

As we learn more about the technological influences on Einstein's life and his manner of thinking, we see that far from the process of pure mentation implied by accounts of his thought experiments and highly mathematical treatises (Stachel, 1982), the references to trains and systems of synchronized clocks were not mere abstractions to justify a formal model (Galison, 2003). As train travel spread across Europe during the early 20th century, the problem of calibrating and coordinating time in different towns became significant and practical. As a patent officer, Einstein was exposed to applications proposing electro-mechanical ways to coordinate clocks and reliably schedule trains. To Einstein, the technological artifacts of the day—systems of electronically coordinated clocks and traveling trains—were powerful "things to think with" (M. Resnick, Martin, Sargent, & Silverman, 1996) and to think about. Thus, it may have been the exposure to these technological influences—to *applied* science—that enabled this major advancement of formal theories of physics. This runs counter to the traditional view of science ingrained in FF, which posits that formal knowledge is fundamentally prerequisite to applied knowledge.

Such counterexamples are abundant, however, and span the range of scientific study. In *Thing Knowledge*, Baird (2004) provided one such treasure trove, reviewing the history of a range of technological advancements that each represent "materialist conceptions of knowledge" (p. 1), rather than formal conceptions, which present theory, offer predictions, and "do epistemological work" (p. 17). One illustrative example is Baird's account of Michael Faraday's material and literary contributions to our basic understanding of the physical world. In 1821, Faraday constructed a device that produced rotary motion by adjusting current that varied a magnetic field in synchrony with the rotor, effectively pulling the rotor forward throughout the cycle. Faraday had invented the electromagnetic motor, a cornerstone of modern technology. In so doing, Faraday revealed fundamental knowledge about all forms of electromagnetic phenomena, including light. He also identified important aspects of the conservation of energy, as electricity was converted to mechanical motion. Notably, he accomplished all this without deriving his design from formal theories or equations. Indeed, the formalized scientific theories and mathematical formulae trailed his discovery by years. To disseminate his scientific work, Faraday actually shipped prebuilt versions of the motor the way scholars today share reprints and digital files of their scientific papers.

As Baird and others (e.g., Cajas, 2001; Meli, 2006) show, technology and applied knowledge can and often does precede the development of formal, scientific theory. Yet the scientific community, and society more generally, seldom give the technological advancements equal billing with codified scientific text and formalisms. Despite evidence of the power of material invention to advance scientific theory, we generally accept the FF view that technological advancements are born from theory, and to be legitimate they must be derived from formal knowledge represented in symbolic and specialized notation.

FORMALISMS IN FORMAL EDUCATION

In education the privileged role of formalisms and formal knowledge plays out along similar lines, as a belief that to learn a specific content area one needs to first master the formalisms for a domain before one can be expected to effectively apply that domain knowledge. Laurillard (2001) made a similar observation in undergraduate education in the United Kingdom but appears to draw on something akin to the *broad view of formalisms* mentioned earlier, when she conceptualized the distinction between formal representations and the things they reference. She included in her account all things that are *descriptions* of primary experiences—those things that mediate our access to the experiences and objects themselves and, consequently, the ways we process them. As with the narrow view, this view, too, acknowledges the abstract nature of formalisms with respect to the phenomena they purport to describe. Laurillard made the observation that the bulk of higher education students' formal education is "acting on descriptions of the world" (p. 55) rather than observing or acting directly on the world. In her view, formal education diverges from the everyday or *practical experiences* of students (Laurillard calls these *first-order experiences*) and focuses instead on *academic* (*second-order*) *descriptions of experiences* and the knowledge and skills students need to use these formal descriptions of the world in scholastic discourse (Gee, 2004). Drawing from the broad view of formalisms, Laurillard (and Gee, 2004, as well) would include in her critique of FF all linguistic and visual forms that mediate access to the directly perceived world, including historical accounts and the use of analytic cases, and formal theories and principles.

Beliefs about the primacy of knowledge of the formalisms of a domain as prerequisite to applied domain knowledge seem to be particularly prevalent in STEM fields and STEM education. It is here where we are most likely to see FF based on the narrow view of formalisms as conventionalized notational systems with no inherent meaning, and it is in these STEM domains where there is a compelling body of evidence that FF is widespread and suboptimal. However, I show support that FF views are present in the humanities and social sciences, as well, where the broader view of formalisms more aptly applies.

As an educational psychologist who studies teaching and learning, I am interested in the implicit beliefs that those of us in education, and in society more generally, have about how people learn—particularly how people develop a conceptual understanding of new content (Kalchman & Nathan, 2001)—and how these beliefs shape our explicit prescriptions for how learners should be taught within formal education settings.[1] The role of formalisms in education has gained little critical attention yet has great implications regarding the aims of public education and the manner in which policies and practices for education are developed and implemented. In particular, if one believes that disciplinary knowledge obtains

[1]The claims in this article are about formal educational settings. Informal and extracurricular education may follow different guidelines (e.g., L. B. Resnick, 1987) and tend to treat formalisms in a different manner (e.g., Rose, 2004).

its structure from a foundation built on discipline-specific formal representations of scientific theories and mathematical laws, it is natural to conceptualize learning and instruction as primarily directed toward acquiring and exhibiting knowledge of those formalisms.

In the remainder of this article, I set out to further articulate the FF view and examine beliefs and practices for the role of formalisms. With math education as my initial context, I show how the FF view is manifest in teachers' beliefs about learning and in curricular organization, though these pedagogical and curricular views actually contradict aspects of students' patterns of performance. I then show that FF can be observed in a wide array of education content areas beyond math, including physics, chemistry, the nursing and engineering professions, language arts, and even teacher education. I argue that the FF view arises out of an apparent conflation between the power and utility of formalisms to do discipline-specific work, on one hand, and the processes by which learners are expected to develop disciplinary competencies in the first place. After identifying some of the more serious problems with an education program that strictly follows a FF view, I review promising research on approaches to learning that support progressive formalization and project-based learning by building upon students' applied knowledge and experiences. I conclude by reflecting on the ways beliefs about knowledge development and learning influence educational practices and characterize the aims of formal education.

THE FORMALISMS FIRST VIEW

Widely adopted accounts of scientific advancement rest on a deep-seated view of the pre-eminence of formalized knowledge, and the derivative nature and subordinate status of applied knowledge (Baird, 2004; Cajas, 2001; Stokes, 1997). In short, FF posits that conceptual development proceeds from the formal to the applied. In this section I examine the sources of support for the FF view and its influence on education. I show how this general perspective appears in different forms across several curriculum content areas and look at its pertinence and limitations for describing student performance and conceptual development.

Support for the Formalisms First View

In Western culture there is a tendency to elevate formal reasoning and "pure" mathematics and science above that of the practical and physical. The philosopher John Searle (1990) placed this view in the context of Mind-Body Dualism, where the mind is regarded as distinct from biological entities, as "something formal and abstract, not a part of the wet and slimy stuff in our heads" (p. 31). Dualism, along with accounts of scientific advancements such as Einstein's discovery, is found throughout the history of Western thought.

Roots of the formalisms first view. Scientific inquiry, from its earliest expression during the classic Greek era, has been separated from practical use. Traditional Western philosophy, from Plato (c. 400 BC) to Hume (c. mid-18th century), considered the "detached theoretical viewpoint" to be superior to "everyday practical concerns" (Dreyfus, 1991, p. 6). Inquiry was elevated to its highest form when it was purely for the pursuit of knowledge and offered no direct application (Stokes, 1997). Plato (trans. 1992) considered the "philosophical arts" in higher regard than the "manual arts." Consequently, the elite of the Hellenic and Greco-Roman eras held the use-oriented engineer and technologist in contempt (Stokes, 1997). Manual labor was relegated to those of lower social status, often slaves. Reflecting on this classic view of scientific practice, Baird (2004) noted,

> To do proper epistemology, we have to "ascend" from the material world to the "Platonic world" of thought. This may reflect Plato's concern with the impermanence of the material world and what [Plato] saw as the unchanging external perfection of the realm of forms. If knowledge is timeless, it cannot exist in the corruptible material realm. (pp. 5–7)

Even in modern times, technology has been viewed as subordinate to pure research, often regarded as an outgrowth or application of scientific knowledge (Barnes, 1982; Cajas, 1998). It was most clearly articulated in a report submitted to the President of the United States entitled "Science The Endless Frontier" in July 1945 by Vannevar Bush, the director of the U.S. Office of Scientific Research and Development (Stokes, 1997). Bush produced a lasting blueprint for America's postwar policies governing science research and for making funding decisions. In statements that clearly resonated with the scientific community at the time, Bush coined the term "basic research" and defined it as research "performed without thought of practical ends" that produces "general knowledge and an understanding of nature and its laws" (Stokes, 1997, p. 3). To Bush, basic research was preeminent, the driving force behind the major advances in technology and the pacesetter of technological progress (Stokes, 1997). When the National Science Foundation was founded in 1950, it adopted this essential view and used it to guide decisions for allocating scarce resources to fund the advancement of science (as cited inStokes, 1997).

Empirical support for formalisms first view. Recent research on learning has provided some empirical support for FF. There are circumstances where realistic depictions detract from the core idea and hamper transfer (like Baird's "corruptible material realm"). In contrast, formal representations of mathematics concepts are detached and "incorruptible," remaining true to the ideas at hand, thereby supporting transfer more readily than concrete instances. For example, when concrete objects and ornate representations are used to stand for things symbolically, their perceptual properties detract from their representational roles. In a rich line of research, DeLoache, Uttal, and colleagues (e.g., DeLoache, 1995, 2000; Uttal, Liu, & DeLoache, 1999) have shown that children struggle with "dual representations" where they must treat entities as both objects and symbolic depictions of objects. Scale models, such as dollhouses or spatial layouts, prove to be *more effective* as representations of the things they are intended to depict when the perceptual salience of the scale models is *reduced*. In a similar vein, McNeil and colleagues (2010) showed that mnemonic abbreviations in mathematics (like *b* for the cost of brownies; see Küchemann, 1978) can hinder students' interpretations of algebraic expressions because their salience invites spurious associations. Those presented with nonmnemonic symbols, such as X and Y or Greek letters, were more likely to provide structural interpretations of the expression.

Another point favoring formalisms is that representations that show less resemblance to the things they are purported to stand for can mediate superior transfer. Kaminski and colleagues demonstrated that when using an abstract or "generic" instantiation of the concept and the rules that govern it, both undergraduate students and young children show superior transfer than those presented with the concept and its rules in concrete form (Kaminski, Sloutsky, & Heckler, 2008). With minimal training (Experiment 1 in Kaminski, Sloutsky, & Heckler, 2009b), kindergarteners exposed to abstract representations of proportions (the proportion of black circles among black and white circles) applied the concept to novel objects with much greater accuracy, whereas those presented with the concrete objects (proportion of images of cupcakes with and without colorful sprinkles and cherries) operated at chance levels. In a second experiment, kindergarteners given the abstract circles were much more successful at applying numerical fraction labels to novel object proportions than those who learned from the concrete instances. Surveying their findings over several studies, Kaminski et al. (2009b) argued that "irrelevant perceptual richness of some concrete instantiations can potentially hinder both learning and transfer" (p. 154; also Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009). On this basis they concluded,

> If a primary goal of learning abstract concepts such as mathematical concepts is the ability to recognize novel instantiations and successfully transfer knowledge, then educational material should maximize the likelihood of attending to relational structure and minimize the likelihood of diverting attention primarily to the superficial. One way of achieving this is to present mathematical concepts via generic formats, such as traditional symbolic notation. (p. 154)

However, as I later address, some scholars have critiqued the theoretical basis of the work as well as its implications for education (e.g., De Bock, Deprez, Van Dooren, Roelens, & Verschaffel, 2011; Jones, 2009).

Formalisms First View in Educational Practice: The Case of Algebra

Studies such as those conducted by Kaminski and colleagues offer an important perspective on the relative strengths and weaknesses of concrete and abstract representations. To gauge the impact of FF on education more fully, we should also look for evidence of its pervasiveness in scholastic settings. If FF is prominent, we can expect to find it shaping the perceptions of teachers, influencing the design of curricula, and predicting the performance patterns of students.

FF views among mathematics teachers. Cobb, Yackel, and Wood (1992) lamented the divisions that students confront between the classroom and out-of-school settings. They noted, "It is only after mathematical structures have been presented to students in finished form that they are taught to apply this knowledge to situations that bear a closer resemblance to those that they might encounter outside the mathematics classroom" (p. 13). To understand the prevalence of the FF view within education, consider data from math teachers asked to make predictions about student performance on problems that were closer or further from abstract, symbolic formats. A series of studies looked at whether teachers preferred a learning trajectory that privileged formal equations as an entry point—consistent with the FF view—or were more likely to favor beginning with more applied, concrete tasks such as story problems before focusing on formalism. In one study (Nathan & Koedinger, 2000c), high school teachers ($N = 67$) were asked to predict how high school students who had successfully completed 1 year of algebra instruction would perform on a set of items that were more abstract (equations) or more concrete (story problems and word equations; see Table 1). The data on student performance are presented somewhat later in this article. Of interest here are the types of items teachers were asked to evaluate. As shown in the examples of Table 1, arithmetic and algebra problems can look structurally similar but differ by the location of the unknown quantity. For algebra problems, students are asked to reason about unknown quantities in relation to other quantities (the second row of Table 1). In contrast, arithmetic problems provide the unknown value by itself (top row).

In addition to the arithmetic-algebra distinction, problems presented to teachers varied in their presentation formats, as shown by comparing across the columns of Table 1. Symbolic problems rely on formal syntax and algebraic notation. Verbal problems, in contrast, use words rather than formal symbol structures and can further be subdivided into story problems that include a situational context and word-equations that verbally describe the relations found in symbolic equations without an explicit context. This 2 (rows)$\times$3 (columns)

TABLE 1
A Sample of the Structurally Matched Problems Given to Teachers to Elicit Their Expectations of Student Arithmetic and Algebraic Problem Difficulty, Organized by the Presentation Format (Columns) and the Position of the Unknown Value (Rows)

	Verbal Problems		*Symbolic Problems*
Presentation format → Position of the unknown value ↓	Story	Word-Equation	Equation
Result-unknown (arithmetic)	When Ted got home from his waiter job, he took the $81.90 he earned that day and subtracted the $66 he received in tips. Then he divided the remaining money by the 6 hours he worked and found his hourly wage. How much per hour does Ted make?	Starting with 81.90, if I subtract 66 and then divide by 6, I get a number. What is it?	Solve for X: $(81.90—66) / 6 = X$
Start-unknown (algebra)	When Ted got home from his waiter job, he multiplied his hourly wage by the 6 hours he worked that day. Then he added the $66 he made in tips and found he earned $81.90. How much per hour does Ted make?	Starting with some number, if I multiply it by 6 and then add 66, I get 81.90. What number did I start with?	Solve for X: $6X + 66 = 81.90$

design allowed Nathan and Koedinger (2000c) to investigate how differences between formal equations and concrete word problems affected teachers' predictions about student performance and whether these differences were evident across both arithmetic and algebraic items. Other than these differences, items with different formats were carefully matched for their underlying quantitative relations.

None of the item distinctions discussed here were made explicit to the teachers during their prediction task, yet teachers were clearly sensitive to them. Eighty-four percent of high school math teachers ranked the arithmetic level problems (top row of Table 1) as being more accessible to students than the algebra (start-unknown) problems. Furthermore, in keeping with the FF view, teachers consistently ranked the symbolic problems as easier for students than the verbal story problems or word equations, regardless of whether they were arithmetic or algebraic. In contrast, only 12% expected that equations would be harder for students than verbal problems. When looking specifically at the algebra items, more than 75% of the teachers expected high school algebra students to perform better on algebra equations than algebra word equations or algebra story problems.

Clearly, teachers shared the FF view: Formalisms were perceived as a natural entry point for algebraic reasoning, preceding competency in solving word problems. Applying algebra concepts to verbal problems was expected to be much less accessible to algebra students. Notably, this general pattern is widespread among practicing teachers (Nathan & Koedinger, 2000a; Petrosino & Gordon, 2011) and preservice teachers (Nathan & Petrosino, 2003; Petrosino & Gordon, 2011) in different regions of the United States.

When asked to justify their predictions to the problem-ranking task (Nathan & Koedinger, 2000a, 2000c; Nathan & Petrosino, 2003), both in-service and preservice teachers stated that symbolic reasoning was more basic and "pure" than the verbal problems and provided the most natural way to introduce arithmetic and algebraic problem solving. Echoing Kaminski and colleagues, one teacher stated, "[The story problem] provides a scenario that seems more likely to distract or confuse students." Teachers also posited that skill in solving equations was a necessary prerequisite for algebra "applications" such as solving story problems.

FF views in algebra textbooks. The FF view is apparently well ingrained in practicing and prospective teachers. Another area of potential influence in education is the design of school curricula. Textbooks influence what mathematics is taught and how it is taught (Huntley, 2008; Jacob, 2001). In a study of a national sample of science and mathematics teachers (Weiss, Pasley, Smith, Banilower & Heck, 2003), researchers found that

> textbooks are second only to teachers' knowledge, experiences, and beliefs in the frequency of influence on instruction. The majority of teachers (71 percent nationally) rely to some extent on the textbook/curriculum program in their school or district in making decisions on how to teach. (p. 93)

Textbooks are important societal artifacts to examine in this regard because they reveal institutionalized views about learning and conceptual development that may be otherwise implicit but are subsequently reified in specific design choices regarding curriculum organization and sequencing.

A corpus analysis of 10 widely adopted commercial algebra textbooks (including more than 1,000 curriculum sections), chosen from the adoptions lists of teachers involved in one of the ranking studies just cited, showed that new material was most often introduced first through symbolic formalisms, later followed by story problem solving as applications of the new material (Nathan, Long, & Alibali, 2002). Eight of 10 textbooks showed a statistically significant preference of presenting problems symbolically prior to presenting problems in verbal form. This was consistent with what the researchers termed the *symbol precedence view* of algebraic development, which, like FF, posits that students cannot gain mastery with applied tasks such as story problem solving *until* they first demonstrate mastery with the formalisms—in this case, algebra equations. The two textbooks that did not show the symbol precedence view were both reform-based texts from the University of Chicago School Mathematics Project series. The authors asserted that "these results establish the strong preferences of the publishers of textbooks in the sample to introduce algebraic activities for new learners in a symbolic form and then move learners on to verbal problems as applications and extensions" (Nathan et al., 2002, p. 13).

Since the publication of that original corpus analysis of algebra textbooks, education reform has grown. Many textbooks now provide contextualized and applied supplementary materials online that may not align with the FF patterns. Although online materials emphasizing applied knowledge may be on the rise, their supplementary status may further affirm the FF view: They are important, but not sufficiently so as to alter the core curricular sequence.

The original textbook study also found that the symbol precedence pattern was more prevalent among traditional math texts and was not more likely than chance to occur in reform textbooks. It is possible that reform textbooks with alternative sequencing patterns now make up a more substantial portion of the market than they did when the initial study was conducted 10 years ago. One relevant issue is the level of market penetration of reform textbooks; if they are radically different but affect a small segment of the market, then the overall impact is likely to be small. Here it is quite difficult to get reliable data as sales and adoption levels remain closely guarded information among publishers. Estimates by insiders put the best selling middle school math curriculum at about one third of the market (Anonymous, personal communication, December 7, 2010), which is substantial but not domineering. Curriculum adoptions at the elementary and secondary grade bands are known to be much more varied, with no one curriculum—reform or traditional—dominating these markets (Madison Metropolitan School District, 2009).

New, and as yet unpublished, research has examined patterns among more recent algebra textbooks. In this study, Sherman (2010) extended the line of research initiated by Nathan and colleagues by looking at both the student textbook organization of written exercises in each section of each book (as done in Nathan et al., 2002) and the *recommended* exercises as directed in the teacher versions of the textbooks. Essentially, the teacher editions guide teachers on which exercises to assign (or do in class) and in which order. Sherman compared "conventional" ($n = 5$) with National Science Foundation–funded, "standards-based" ($n = 5$) curricula spanning from 1995 to 2008. Two of his findings are relevant to the current discussion. First, reform textbooks in the newer sample did not exhibit the symbol precedence pattern, showing they are not guided by the FF view, replicating Nathan and colleagues' earlier finding. Second, conventional textbooks operating in the current standards-based climate fail to show the symbol precedence pattern in the student textbooks (at odds with the older study) but *do* show the symbol precedence pattern significantly above chance levels when considering the *recommended items* in the teachers' editions. Sherman's analysis noted that student editions seem to "front" the exercises with verbal and applied problems, but these are actually passed over or targeted for later consideration in the publishers' teacher editions. Sherman concluded,

> Thus, while the structure of the written exercises as they appear in conventional student texts seems to have changed over the last ten to fifteen years as described above, a symbols-first approach is still prevalent with respect to how authors of these curricula intend them to be used by teachers. (p. 27)

Taken together, these studies of math textbooks suggest that reform movements influence curriculum designs, yet FF views still persist and remain widespread.

FF views among algebra education researchers. Although mathematics teachers and textbook designers may operate with arcane views of learning, mathematics education researchers are well informed by contemporary learning theory. If researchers exhibit patterns such as the symbol precedence view found in teacher responses and textbook organization, this suggests that such views are deeply entrenched within the education community.

An investigation of researchers' views (Nathan & Koedinger, 2000c) used the ranking task presented to teachers. The researchers who participated ($N = 35$) were all actively engaged in scholarship regarding algebra learning and instruction. They were dispersed throughout the world and were all members of the Algebra Working Group of the Psychology of Mathematics Education professional organization, through which they were recruited.

As with the teachers, a majority of the algebra researchers (66%) ranked start-unknown (algebra) problems as consistently more difficult for students than arithmetic problems. Researchers also ranked symbolic equations at the arithmetic level and the algebraic level as easiest for students 31% of the time—giving it the strongest consistent ranking among the six problem types—and nearly half supported a developmental trajectory placing equations as the easiest among algebra problems. Looking across the teacher and researcher data, the authors concluded that "teachers and researchers who are examining a problem for its level of difficulty make their decisions on the basis of the question 'How far along the developmental trajectory from symbolic arithmetic to algebra story-problem solving has a student progressed?'" (p. 182).

FF patterns in algebra student performance. Having observed the prevalence of the FF view among math teachers, algebra researchers, and in algebra curriculum design, it is clear that FF has a notable presence in math education. However, if FF describes students' algebraic development, then students should show a performance advantage for formal, symbolic representations early in their algebraic reasoning, prior to demonstrating competency in applied tasks like story problem solving.

Performance on items that fit the six categories in Table 1 has been examined in a series of experimental studies. Across the board, researchers have found that arithmetic problems are easier than algebra (start-unknown) problems (Koedinger & Nathan, 2004; Nathan & Kim, 2007), as expected from earlier research (Carpenter & Moser, 1983). A more direct test of the FF view, however, compares performance on formal equations with that of mathematically matched word equations and story problems. In one study (Koedinger & Nathan, 2004, Experiment 1), algebra students in an urban high school ($N = 76$) showed superior performance (about 64% correct) solving the verbally presented story and word-equation problems than symbolic equations, a finding that runs counter to that predicted by FF. The higher performance on verbal problems has been replicated in several other studies, including those with samples of other high school students ($N = 171$ in Koedinger & Nathan, 2004, Experiment 2), middle school students ($N = 90$ in Nathan et al., 2002; $N = 372$ in Nathan & Kim, 2007), remedial community college ($N = 153$ in Koedinger, Alibali, & Nathan, 2008), and high-performing university students ($N = 65$ with mean math SAT score of 719 in Koedinger et al., 2008) among others (see Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; and Weinberg, 2004, for additional replications.)

Despite formal instruction in prealgebra and algebra, high school and college students still often do not know what to do with symbolic equations. Across these studies, students frequently gave *no response* to symbol problems (high school students gave no response to 32% of symbolic equations, double that of other representations), suggesting a basic failure to comprehend the meaning of the formalisms (Koedinger & Nathan, 2004). When students directly solve equations, their attempts are highly error prone and their likelihood of success significantly lower than for the strategies elicited by word equations and story problems.

Contrary to the expectations within the FF view, a developmental analysis of student performance showed that it was extremely rare for a student to solve arithmetic and algebraic equations without also demonstrating success solving verbal problems (only 1 in 76 students in Experiment 1, and 1 out of 171 in Experiment 2 showed this pattern; Nathan & Koedinger, 2000c). Furthermore, the developmental analysis showed that algebra students were far *more likely* to solve symbolic problems correctly if they *also* correctly solved word equations and story problems. Together, these findings support a developmental trajectory that places primacy with applied story problem solving before demonstrating competency with algebraic formalisms—in stark contrast to the FF views evident among teachers, researchers, and curriculum designers.

Prevalence of the Formalisms First View in Other Content Areas

The symbol precedence view in mathematics education is one of many approaches that fit within the formalisms first view of conceptual development. If this were its only manifestation, FF would be notable for math education but of little importance to education more generally. In this section, I set out to show that FF patterns of curriculum design and instruction can be found throughout education. In science education, for example, formal symbols and models are often taken as primary, and their understanding as essential for students' later success with applied problem solving, as well as the transfer of that knowledge for understanding technology, engineering, and other scientific application areas (e.g., Bloomfield, 1998/2004; Bond-Robinson, 2002; Cajas, 1998, 2001). Up until now, the phenomenon has gone largely uninvestigated, yet there is a convergence of evidence across a number of content areas worth examining, including analyses of textbook organization and curriculum sequencing, student performance measures, and student responses to surveys.

FF views in physics textbooks. Physics is an area where the preeminence of symbolic representations is well established. A comparative analysis of two college-level introductory physics textbooks (Nathan, 2003) highlights the FF approach as it plays out in curriculum organization. In one of the most widely adopted, traditional textbooks in the United States (Giancoli, 2004), physics topics were examined for the conceptual chain leading up to the focal topic for a unit. The pattern is consistent with FF. For example, the topic of hydraulics begins with the introduction of the concept of density (D), which is presented first in narrative form and then as the formal relation between mass (m) and volume (V), $D = m/V$. The instructional sequence then moves from density to the concept of pressure, represented analytically as the amount of force (F) per area, (A), by the equation $P = F/A$, and, diagrammatically, as an idealized cube of fluid with arrows (idealized forces) pointing in from all sides. Pascal's Principle is then given, noting that the pressure applied to a confined fluid increases the pressure throughout the entire sample by the same amount (Giancoli, 2004, pp. 279 ff.). Next, the concept hydraulics is discussed, first quite broadly and then, by example, as the underlying principle that describes hydraulic lifts, brakes, and the function and design of large hydraulic elevators. Hydraulic elevators are presented as the outgrowth and application of scientific theory of the science of hydraulics. This analysis reveals a general structure starting with formal symbols and equations, moving on to worked examples and illustrations of idealized principles, followed then by the application of the formalisms to technological innovations. The pattern, prototypical of the FF approach, is common throughout this and other popular and influential textbooks, such as *Sears and Zemansky's University Physics with Modern Physics* (Young, Freedman, & Ford, 2007), which is now in its 12th edition.

In contrast, a less widely adopted cadre of reform textbooks uses application areas to motivate physics instruction. Typically in this approach, the author introduces one or more technological innovations or application areas and shows how the behavior of these various applications relies upon common scientific principles. The volume that was analyzed is Bloomfield's (1998/2004) college textbook, *How Things Work: The Physics of Everyday Life.* Here, the topic of hydraulics appears in a chapter on elevators. The conceptual chain leading up to this chapter starts with the behavior of water faucets and water hammer (the loud noise you sometimes hear in your water pipes), vacuum cleaners, airplanes and Frisbees, bicycle gears and freewheels, brakes, pneumatic tires, fuel tanks, pumps and turbines. Hydraulics and the physical laws of pressure are identified as common principles across a range of technologies.

Of interest, the aims of these two approaches to physics instruction have much in common. For example, the quantitative problems at the end of the respective chapters are quite similar. Yet they embody sharply contrasting underlying models of conceptual development—the trajectory by which students are expected to attain these goals. In the FF trajectory in the traditional textbook, students first learn general laws of fluids steeped in the formalisms of algebraic equations and then learn how the laws can be applied to specific problems and devices. The emphasis is on *deductive* learning and application of formalisms to concrete situations. In the alternative approach, students first learn about the behavior and design of specific, familiar devices that exemplify a common set of physical relationships. The physical laws arise as generalizations of common principles that model (or govern) these behaviors. The relevant formalisms are then developed *inductively*, as instantiated across a range of applications.

Student performance data in physics. With FF-based curricula, what is the impact on student performance? In a study of beginning college physics students conducted over 5 successive years ($N = 408$, with annual sample sizes ranging from 74 to 96), Meltzer (2005) looked at students' facility with *vector diagrams*, which he compared with performance on matched verbal problems. Although arrow diagrams[2] and other visual representations can exhibit concrete qualities, vector diagrams are a specialized notational system that fit within my definition of formalism in that they employ conventions to map generic objects (circles and arrows) to represent any of a number of possible entities (e.g., electrons, planets) and forces (gravity, electromagnetism) as described in a range of physics problems. Meltzer found that students were more likely to correctly solve conceptual, decontextualized physics multiple-choice problems involving Newton's Third Law ("For every action, there is an equal and opposite reaction") when those questions were presented verbally, without the formal notation, as opposed to using matched items with vector diagrams. Meltzer found that error rates were significantly higher on the formal versions of the problems both at pretest and on students' final (end-of-course) examinations. A significant disadvantage for formalisms-based problems was found for the population as a whole. It even held for the subset of students enrolled in the calculus-based physics course ($n = 240$), even though these students had greater mathematics training than the sample as a whole and their relative performance levels on each item were much higher than the sample means.

Similar results were reported by Heckler (2010). Introductory college physics students showed significantly lower performance when they were prompted to solve standard problems using a formal force diagram method compared to students with no prompting. Analyses of solution strategies and errors revealed that, when using force diagrams, users made errors that revealed their poor understanding of the formalisms themselves, whereas students in the control group used intuitive methods that were more meaningful and more reliable. In Sherin's (2001) terms, the students in both the Meltzer and Heckler studies failed to understand what the formalisms "say." Curricula may follow the FF approach, but if formalisms are poorly understood, then little advancement toward problem solving and other applications can be expected.

FF view in chemistry instruction. Chemistry is a discipline deeply rooted in empirical inquiry. Yet here, too, the influences of FF are evident. An analysis of a typical undergraduate, general chemistry textbook revealed that the underlying model of learning was based centrally on the assumption that all students understood the symbolic notation of chemical elements and processes as prerequisite to "doing" chemistry (Bond-Robinson, 2002). Half of all topic explanations required understanding some form of symbolic notation. Of those, 60% required understanding symbols and notational conventions unique to chemistry, whereas 40% required understanding of mathematical symbols to make the explanations comprehensible. Only 6% of topic explanations in the textbook used primarily verbal descriptions.

Students' views of FF in chemistry education. How do undergraduates respond to this formalism-heavy approach to chemistry education? Research findings show that learners struggle with information presented through formal notation. A study using structured surveys of undergraduate science majors ($N = 600$) in their first semester of chemistry, along with interviews of their teaching assistants, revealed

[2]In unpublished research, Kurt VanLehn found that participants in his studies on learning systems dynamics treat stock and flow diagrams as abstract representations rather than in a concrete or iconic manner (personal communication, June 28, 2011).

that students had tremendous difficulty understanding and properly using formal symbols to stand for chemical elements or to model chemical processes (Bond-Robinson, 2002). Students rated formal explanations at the atomic level as less useful than explanations given at the more applied level. In the curriculum, atomic theory and formal chemistry notation were generally presented as precursors to chemistry applications, yet students overwhelmingly reported that these formal representations and principles interfered with their ability to understand the later material.

FF view in professional education. Formalisms are also commonly used in professional education and appear at times to serve a role that is not merely a formalisms-first view but a more stringent *formalisms-only* view of learning. For example, novices in nursing encounter formalisms when they are trained to represent complex procedures and policies. In a study of more than 100 nursing students and expert nurse practitioners, Benner (1984) showed how formalisms (such as premedication procedures and formal pain rating) were often seized upon by novices because they recognized the formalisms as a fundamental part of their training. Yet Benner reported that the formalisms often obscured learning and interfered with novices' and advanced beginners' abilities to carry out the practices correctly. For example, nursing students and instructors frequently encountered knowledge and practices that could not be expressed with existing formal models, theories, or representations. Likewise, formal knowledge did not translate into practical actions that nurses needed to take in order to capitalize on theoretically postulated input–output relations (Benner, Tanner, & Chelsea, 2009).

The study of nursing education has also documented a climate that devalued knowledge and practices that could not be formalized (the formalisms-only view), regardless of their effectiveness for treating patients. An additional study of the role of formal models and representations in functioning surgical units (Gordon, 1984) revealed other shortcomings of the formalisms, such as medication flow rates, used in clinical practice, including that novice nurses would inadvertently equate formal models with reality, rather than seeing them as representations of reality (reification), and that formalization led to overconformity as diverse concepts and procedures were forced into the same categories in order to fit with the prevailing formalisms. In reevaluating the state of clinical training, Benner and colleagues (2009) have noted most recently, "Classroom presentations of 'nursing knowledge, science, theory, and technology' have been assumed to be the 'blueprint' or abstract knowledge to be literally 'applied' in the clinical setting" (pp. 383–384) even while the abstractions fall demonstrably short in accurately describing the clinical situation and guiding clinician's behaviors.

The FF issue has long been a source of concern for professional training programs more generally. The Nobel laureate Herbert Simon (1969/1996) observed in the early post-Sputnik era that "engineering schools gradually became schools of physics and mathematics; medical schools became schools of biological science, business schools became schools of finite mathematics" (p. 111). As Cajas (1998) noted, this was still true 30 years later:

> The way in which future technologists (e.g., engineers or medical doctors) are generally prepared is the following: Students first take science classes with the assumption that such classes can be applied to specific technological problems (e.g., engineering problems, medical problems). The justification of taking science classes (physics for example in the case of engineers or physiology in the case of medicine) is that these classes are the bases of their future professional work. (p. 5)

Engineering education commonly reveals this general FF approach, even as it goes through substantial reform. The centrality of formal math and science is well established in engineering (National Academy of Engineering, 2010; National Research Council, 2006), but its developmental primacy for beginning engineers is presumed. Professional training programs typically require that students exhibit mastery with formalisms from these content areas before they gain access to discipline-specific studies (mechanical, chemical, electrical, biomedical engineering, etc.) or to the rich design activities that make up much of advanced engineering studies and workplace practices (Guzdial et al., 2001; Klingbeil, Mercer, Rattan, Raymer, & Reynolds, 2004; McKenna, McMartin, & Agogino, 2000). Design is frequently regarded as "both the cornerstone and capstone of the engineering curriculum" (Sheppard, 2001, p. 440). Yet one of the barriers for learning design, as noted by Sheppard (2001), is the early and continued dissociation of formal methods of analysis from engineering design. In the current reform climate, some have successfully argued that the 1st-year experience for engineering students should be built around project-based design activities (Ambrose & Amon, 1997; National Academy of Engineering, 2005; Sheppard & Jenison, 1997a, 1997b), though this is being adopted in a limited fashion.

Even within this reform climate, analyses of high school curricula and classroom instruction, admissions requirements for university programs, and most freshmen experiences still structure engineering education from within an FF framework (Prevost, Nathan, Stein, Tran, & Phelps, 2009; Stevens, O'Connor, & Garrison, 2005). For example, research from the Center for the Advancement of Engineering Education (Stevens et al., 2005) reports that 1st-year engineering students typically must "prove themselves" through a series of mathematics and science courses that present the "foundations" of engineering (e.g., calculus, physics) far removed from the workplace practices of professional engineers. Thus, even in STEM training programs primarily designed toward application, the FF approach of learning and instruction is highly influential.

FF view in the social sciences. Because of their close ties to formal models of the physical world, it might be expected that formalisms play a central role in learning and performance within the STEM fields. Yet formalisms are also prominent in the social sciences. For example, economics students regularly encounter the simultaneous presentation of multiple formal representations and must learn to flexibly coordinate graphs, tables, and equations to address substantive questions about consumer preferences and the changing marketplace. Experts in economics do this coordination regularly and fluently. However, H. J. M. Tabachneck (1992; H. J. M. Tabachneck, Leonardo & Simon, 1994) showed that novices experience great difficulty using the individual representations that are formally presented in economics textbook problems and perform quite poorly with more demanding forms of reasoning that frequently require them to coordinate multiple representations. Verbal reports taken of beginning economics students at the college level show they make incorrect inferences about formal representations, such as graphs of supply and demand, even when direct perception would do. Students also form individuated mental representations from the given formal representations, such as tables, graphs, and systems of equations that are rarely integrated across the formal representations, and, when they are combined, they are most often combined incorrectly. This manner of instruction proves to be highly problematic as many basic concepts within introductory economics such as supply–demand relations, and the impact of raising taxes on prices are framed in terms of combined formalisms such as symbolic and graphical representations.

Language arts instruction. Language arts, traditionally an area in the humanities, seems remote from the study of the natural and the social sciences. Yet if we apply the broad view of formalisms, we can see that Grossman's (1990) comparative case study of six teachers during their 1st year in high school English classrooms revealed ways that the FF approach was present among the literature and language experts she studied, as when teachers required students to demonstrate mastery of formal grammatical rules before applying them to autobiographical writing. FF among subject matter experts-turned-teachers was evident from convergent sources, including teacher and student interviews, analyses of lesson plans, interviewee performance during structured tasks relevant to classroom planning, and classroom observations. These teachers were compelled to present English and literature as formal disciplines, rather than reconceptualizing English literature and language as school subjects that needed to be taught to novice learners (Grossman, 1989). These teachers operated with an FF view and saw formalisms such as the rules of grammar as prerequisite to writing about one's personal experiences; they also positioned mastery of highly technical literary techniques (such as mimesis and intertextuality) as the gateway to understanding and enjoying literature. In contrast, the teacher education graduates in the study who had less training in literature and linguistics than the content experts directed the focus of literature instruction primarily on the relationship of the student to the text and not exclusively on the text itself. In writing instruction, these teachers identified the standardization of grammar rules as a means to enhance one's ability to communicate with readers and resolve ambiguity rather than as a formal system that dictated writing norms. As in Bloomfield's (1998/2004) application-driven physics course, the end product (e.g., the written essay) motivated the need for formal, linguistic knowledge, and teachers used students' intuitive understandings as an entry point for developing the formal knowledge that is privileged by the discipline.

Conflating Domain Practices With Development

It is not my intention here to globally challenge the power and utility of formalisms for modeling and problem solving. My focus is directed at learning experiences that provide novices the greatest benefit. Formal representations such as equations and graphs are vitally important because of the central organizing role they play for a given discipline. Formalisms support computational efficiency and can mitigate ambiguity (e.g., H. T. Tabachneck, Koedinger, & Nathan, 1995). In addition, formalisms such as symbolic equations can afford superior performance over alternative strategies when solving certain problems, because formal representations often scale up to increased complexity far better than do many idiosyncratic methods (Koedinger et al., 2008). Formal representations can also reveal the common deep structure of quantitative and qualitative relations of seemingly disparate phenomena (such as the relation between mechanical systems and electrical circuits), and thereby provide important conceptual bridges to support transfer, discovery, and theory building (e.g., Goldstone & Son, 2005; Judd, 1908; Kaminski, Sloutsky, & Heckler, 2008; Son & Goldstone, 2009).

These are properties of formalisms that serve the needs of individuals and professional communities with competence in their respective fields. However, there is an apparent conflation of the structure of a discipline and the developmental trajectory by which newcomers gain mastery of that discipline. The FF view uses the disciplinary structure as its developmental roadmap: What is foundational to the discipline is also deemed developmentally primary; what constitutes secondary and peripheral topics to the field then follow in the learning experience; and applications of disciplinary knowledge to practical problems comes last in the scientific process, and therefore are expected to occur later developmentally.

In one particularly notable example of this conflation, the foundational role of formal set theory as the (relatively newly anointed) theoretical basis of numbers and operations in modern mathematics was enormously influential in shaping the design of the original "New Math" program for elementary school instruction of the mid-20th century. However, as we see, it proved to be inadequate for learners and teachers (e.g., Kline, 1973). In the 1950s, the state of mathematics achievement and instruction in the United States was scrutinized by the National Council of Teachers of Mathematics (NCTM) in its Second Report of the Commission on Post War Plans, as well as by the University of Illinois Committee on School Mathematics and the College Entrance Examination Board Commission on Mathematics (NCTM, 1945/1970b). The declining enrollment and waning interest toward mathematics education that began prior to WWII continued, despite the growing importance and marketability of a technical education. By the mid-1950s, the popular press of the time, along with many university mathematicians, declared that the content of K–14 mathematics education had been led by professional educators for too long, with insufficient progress. To turn this tide, academicians turned their attention to school curricula (NCTM, 1970a, p. 76). The fix, they reasoned, was to base mathematics education on the same foundational concepts that were being used to organize the area of mathematics for university study—set theory and number theory. In 1958 the NCTM, along with the Mathematical Association of America and American Mathematical Society, funded the School Mathematics Study Group (SMSG) to produce curricula based on this new conceptual structure. Led by mathematicians, the SMSG is often regarded today as the face of the "New Math" movement. The group was very productive in generating curricular outlines and guidelines, and in producing surveys, evaluations, sample textbooks, and enrichment materials that served as a guide for commercial textbooks across the K–12 grades for many years to follow.

The experts who led the New Math movement believed that the logical foundations of mathematical structure would be transparent to children and, given opportunities, children's understanding would naturally follow (Klein, 2003). Critics, such as the late mathematics professor Morris Kline of NYU (Kline, 1973) and others (e.g., Ahlfors, 1962; NCTM, 1970a), argued that New Math pedagogy was poor and often absent, that the curriculum did not motivate students; that it neglected areas of application, that the curriculum did not promote active participation by students, and that it failed to develop students' intuitive notions of mathematics. Kline criticized what he saw as an overemphasis on the formal structure and notation of set theory. He disparaged the lack of empirical evidence of the efficacy of the new program on measures of student achievement and forms of mathematical reasoning, attitudes toward math, retention, and later interest in math-related fields. He also criticized the lack of staff development for teachers, noting that teachers needed to be better informed about the new content associated with modern mathematics as well as the new program's curricular structure and goals.

Though a formal program evaluation was never conducted, Kline (1973) argued that a there was a proxy evaluation in the form of comparative test scores from the 1964 International Study of Achievement in Mathematics. Because the New Math program was, by this time, ubiquitous in the United States but many other participating countries were still using traditional curricula, Kline suggested that this provided some indication of its success. Although several different grade-appropriate tests were employed, the United States faired poorly in all of them, particularly among the 13-year-old group, which ranked at the bottom. Some of the key developers of New Math had also publicly expressed doubts, with Professor Beberman wondering aloud why they had chosen to put so much emphasis on rigor (i.e., proof), and Professor Edward G. Begle noting, "In our work on curriculum we did not consider the pedagogy" (as cited in Kline, 1973, p. 110).

The New Math program failed, not simply because it offered a poor curriculum but because the mathematicians who organized SMSG did not know a lot about kids or teachers. The mathematical content that formed the basis of New Math had been designed by mathematicians to highlight the formal structure of modern mathematical, with little regard to how that content was to be learned by children, understood by teachers, or taught in classrooms (Klein, 2003). Despite this rocky beginning, many of the basic premises of a program of mathematics education guided by the formal structure of the field of mathematics are still firmly in place in contemporary math education.

The FF view operates broadly and tacitly to influence the design and delivery of learning environments in ways that can be incompatible with students' early skills and emerging developmental needs. The primary role formalisms play in codifying and objectifying the knowledge and practices of a given discipline appears to have been appropriated wholecloth, and without question, by many instructors and curriculum designers as the account of conceptual development. As a lesson learned from the New Math movement, the educational community needs to be careful to distinguish between conceptual structure, as it appears to disciplinary experts, and Bruner's (1960) account of structure as that which makes salient the relations among seemingly unrelated things for the purpose of transfer. "If earlier learning is to render later learning easier, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible" (Bruner, 1960, p. 12). In education, we need to identify and implement curricula and instruction that are not merely true to the disciplines from which they come, but also developmentally "true" to new learners who are engaging with the ideas and discourse practices of a new field (Barab & Roth, 2006).

Problems With the Formalisms First View

By now I have reviewed several ways that the FF view, drawing on both narrow and broad views of formalisms, is in evidence in educational settings, along with evidence of some of the strengths of formalisms, as well as some of the limitations. Yet the picture is not clear-cut, and research findings like those presented by Kaminski and colleagues

(2008, 2009b) and the ensuing critiques of that work are important to understanding the issues formalism use raises. Specifically, Kaminski and colleagues have argued that their research shows that initial mastery of formal representations of abstract concepts and relations prior to applications of those concepts is better suited to support transfer, whereas realistic depictions with perceptually rich features irrelevant to the task hinder learning and transfer. In this and the next section, I lay out some of the shortcomings of those studies.

First, in many of Kaminski at al.'s studies the stimuli that are used to stand for the concrete cases—digital drawings of cupcakes or glasses partially filled with liquid—are themselves still abstractions. As abstract depictions of actual objects they have some of the objectionable qualities of concrete objects, such as task-irrelevant features, but are without many of the multimodal qualities that benefit objects. Martin (2009; Martin & Schwartz, 2005) has shown that an important aspect of successful mathematical problem solving with real objects in areas like proportional reasoning involves touching, shifting, and physically regrouping the objects. Martin shows how actions coupled with interpretations serve as developmental precursors to general mathematical procedures, which can later be enacted mentally.

A second point is to clarify that, whereas concrete representations have "irrelevant perceptual richness" when they are used to serve strictly mathematical purposes (Kaminski et al., 2009b, p. 153), abstract entities, such as equations, have these qualities as well (Kirshner 1989). For example, Landy and Goldstone (2007, 2010) have shown that solvers are influenced by perceptual features of arithmetic and algebraic expressions, such as symbol spacing, which is, technically, irrelevant to syntactic parsing. These perceptions can override well-established formal rules of symbol manipulation, such as order of operations, even when students familiar with the proper rules have recently been reminded to use them.

A third point addresses transfer. Real objects (or their depictions) may hamper learners' ability to easily extract the elements of a core, abstract concept that enable the more distal goal of transfer (Kaminski et al., 2009b). However, in the broader educational context, an important facet of transfer involves learning how to recognize the applicability of a core concept for entities that display a field of task-irrelevant features (Nathan, 1998). When the intended application areas are perceptually rich environments, a substantial part of the skill of learning to solve real-world problems through the application of a previously learned concept is to identify the deep structure that might be obscured by its visual or tactile trappings. Developing one's intellectual abilities exclusively on stripped-down formalisms without exposure to perceptually rich stimuli robs learners of opportunities to learn how to recognize deep structure and filter out irrelevancies. Learning that is steeped in the FF approach cannot develop that skill. As we see, there are alternative approaches that draw on perceptually rich, concrete materials that can support the development of this important aspect of transfer.

Others have also raised important issues concerning transfer in Kaminski et al.'s (2008) original study. Jones (2009) brought up two issues: that the hinting procedure they used is known to strongly affect transfer performance, and although the hints are controlled across conditions, they nevertheless call into question just how much transfer either condition supports; and that the generic (abstract) condition tasks can be seen as closer to the transfer task, along some relevant dimensions. This latter point is central to an empirical study by De Bock and colleagues (2011). They provided a replication and extension of the Kaminski et al. (2008) study. They showed the same transfer advantage to the abstract domain from generic representations. However, they were also able to show greater transfer to a new, concrete transfer task by those using the concrete representations.

A fourth issue is motivational. Students appear to have limited patience with instruction heavily steeped in formalisms (Bond-Robinson, 2002). In relatively short order, students become disengaged. Generally, they favor verbally and visually rich, concrete curriculum materials. The immediate implications are unclear, but attraction and retention in mathematics, and STEM more generally, suffers from a general problem of lack of interest and engagement (NRC, 2005).

Finally, a skeptical account of FF must also acknowledge that there are fundamental concerns about whether (and if so, how) formalisms are valid accounts of the phenomena they purport to model. Scholars in the natural and social sciences have identified fundamental ways in which formalisms fall short. In the natural sciences, Cartwright (1999) showed the limits of scientific fundamentalism, the view that the dominant scientific theories of the day cover all natural phenomena. Rather, her analyses show how scientific "laws" of physics are only applicable under highly constrained conditions. The apparent regularities come about only through what Cartwright terms a *nomological machine*: "a fixed (enough) arrangement of components, or factors, with stable (enough) capacities that in the right sort of stable (enough) environment will, with repeated operation, give rise to the kind of regular behaviour that we represent in our scientific laws" (p. 50). A classic example is the models of motion for dropping a cannonball from the Leaning Tower of Pisa. Yet work with real-world phenomena demands such a degree of modification as to render the laws nonuniversal. As Cartwright pointed out, the law for a falling cannonball simply does not apply to the trajectory of a falling banknote, which is subject to air resistance, more complex hydrodynamics, and the influence of random wind variation. In a similar vein, psychologists Cheng and Holyoak (1985) have shown that there are common forms of reasoning that are not the outcome of applying syntactic rules. Participants appear to draw from pragmatic reasoning schemas, such as notions of permission, causality, and evidence, based on rules that are simultaneously partly generalized and partly context sensitive.

Taken together, this discussion makes clear that issues of representations and transfer are complex (Kaminski, Sloutsky, & Heckler, 2009a) and there is a great deal that is still not known about engineering the conditions that are most favorable for transfer to occur.

PROGRESSIVE FORMALIZATION AND OTHER ALTERNATIVES TO THE FORMALISMS FIRST VIEW

Based on the preceding account of the philosophical and historical bases of the FF view, its prevalence and wide-ranging influence in contemporary education, and its apparent misalignment with learners' trajectories of conceptual development, I argue against instruction and curriculum design that adheres to the FF position. Introducing novices to a new concept or domain of study through formalisms has appeal because it taps into the established knowledge of those who are already highly competent with the content and conventions of formal notation. The previous discussions show that FF is inappropriate in two ways. First, the FF view provides an inadequate mode of conceptual development. Evidence of this comes from examining the curricular sequencing in areas of mathematics, the natural sciences (chemistry and physics), social science (economics), and language arts. In some cases there is evidence that FF views contradict student performance patterns. Even among reform curricula that seemingly break from FF, teacher editions can draw the enacted curriculum toward FF methods. Second, FF encourages a formalisms-only mind-set. This was particularly evident in professional training programs in engineering and nursing, where the emphasis is on hands-on practice. But it was also apparent in the New Math movement, leading the notable scientist-pedagogue Professor Richard Feynman (1965) to raise the serious objection that because the New Math texts were written by pure mathematicians who were uninterested in the connections of mathematics with the real world, that even at the secondary level the program did little to relate mathematics to science and engineering. Despite its ubiquity, FF is an inaccurate portrayal of learning and serves as a poor guide for curriculum design and instructional practice. In the remainder of this section I explore the evidentiary base for alternatives to the FF view. In this and the final section, I consider what these findings imply for instruction.

A central aspect of learning, particularly for conceptually oriented content (as opposed to learning certain procedures), is the extent to which learners connect new knowledge to old and meaningfully relate specialized notational systems (or descriptions of first-order experiences; Laurillard, 2001) to the objects and events in the world that they are intended to represent (Bransford, Brown, & Cocking, 2000; Palmer, 1978). One explanation for students' poor understanding of formalisms across a range of fields is that students may not achieve a *grounded* understanding that allows them to construct meaning of these formalisms in terms of other things that they already understand, or things they can perceive and physically manipulate (Goldstone, Landy, & Son, 2008; Martin, 2009). The computational properties of formalisms, such as equations, come about because of the syntactic, form-based (i.e., *form*-al) rules that govern the relations and transformations of these systems of notation. Yet when formal representations are understood exclusively by reference to other formal representations in the form of rules and mappings, it can lead to an ungrounded form of understanding that appears to be rote, shallow, and rigid (Harnad, 1990; Searle, 1980). A poignant illustration was offered by the philosopher John Searle (as cited in Cole, 2004):

> Imagine a native English speaker who knows no Chinese locked in a room full of boxes of Chinese symbols (a data base) together with a book of instructions for manipulating the symbols (the program). Imagine that people outside the room send in other Chinese symbols, which, unknown to the person in the room, are questions in Chinese (the input). And imagine that by following the instructions in the program the man in the room is able to pass out Chinese symbols, which are correct answers to the questions (the output). The program enables the person in the room to pass . . . for understanding Chinese but he does not understand a word of Chinese.

Although the Chinese Room metaphor was introduced to question the strong claims of artificial intelligence and the prospects that computer programs could someday think like humans, its value in this instance is to demonstrate the challenges of deriving meaning from a system (or room) that only supports formal structures and rules (the program) of formal symbol manipulation. In this case, because many of us would conclude that the person in the room does not understand Chinese, despite the *appearance* that the person understands Chinese, we can similarly question one's understanding of any domain that is rooted exclusively in the use of formalisms.

The contrasting argument is that meaning comes ultimately through reference of formalisms to grounded, nonsymbolic entities such as perceptions, actions, objects, and experiences from the world (Barsalou, 2008; Glenberg, 1997, 1999; Harnad, 1990). Indeed, the claim has been made that some of the advancements in mathematics—such as set theory, the creation of negative numbers, logic, and infinity—can be traced to the physically grounded behavior of objects and events in the everyday world (Chiu, 2000; Johnson, 1999; Lakoff & Johnson, 1999; Lakoff & Nunez, 2000). Although the influences on these historical discoveries is difficult to prove, there is both behavioral and neuroscientific evidence that elements of mathematics, such as numerical reasoning, invoke visual attention (Dehaene, 2011; Fischer, Castell, Dodd, & Pratt, 2003), spatial systems (Dehaene, Bossini, &

Giraux, 1993; Goebel, Walsh, & Rushworth, 2001), and motor action (Badets & Pesenti, 2010; Fischer, 2004), whereas the strength of the relationship between perceptual and motoric systems and number processing reliably predicts math achievement scores (Fayol, Barrouillet, & Marinthe, 1998). It is through grounded relationships that connect to our direct physical and perceptual experiences (or through chains of relations that connect to things that connect to our experiences) that these formal entities attain their meaning. Once meaning is established, however, it is the abstract and form-based properties of formalisms that imbue them with capabilities for quantitative modeling across a broad range of domains, as well as high-speed and high-capacity computation. Formal systems are powerful culturally established tools for advancing our reasoning capacity and for institutionalizing cultural knowledge. Yet they need to be mapped to the world they purport to model to be meaningful and valid (Palmer, 1978).

Even in the face of this, some scholars (e.g., Laurillard, 2001) have argued that scholastic learning environments seldom provide a balanced or integrated education with regard to grounded and formal experiences. Ultimately, both practical and academic experiences seem necessary for a well-educated populace. Instead, historical and policy-based biases favor second-order experiences, particularly as the level of education increases. Yet schools can do more to provide rich, lived experiences that connect to formalized knowledge (Barab & Roth, 2006).

Methods for achieving such balance are founded, in part, on a growing evidentiary base of the study of *progressive formalization* (PF) methods (Romberg, 2001). In both laboratory and field-based settings, PF provides early experiences with first-order, concrete entities to support highly accessible entry points for initial learning and provide the grounded symbolization that fosters the ascension of meaningful formal representations for later generalization and transfer (Abrahamson, 2009; Goldstone et al., 2008; Lesh & Doerr, 2003; Nathan & Koedinger, 2000b). Bloomfield (1998/2004), author of the popular application-driven collegiate physics textbook, makes an insightful comparison to the dominant FF approach when he stated, “While a methodological and logical development of scientific principles can be very satisfying to the seasoned physicist, it can appear alien to an individual who isn’t familiar with the language being used” (p. vii).

Several alternative curricular innovations fall under the general category of PF (Bransford & Schwartz, 1999; Gravemeijer, as cited in Romberg, 2001; Schwartz & Bransford, 1998; Schwartz & Martin, 2004). Romberg (2001, p. 3) stated the overarching design approach quite clearly.

> Rather than starting with the presentation of formal terms, signs, symbols, and rules and expecting students to use these to solve problems (too commonly done in mathematics classes), activities should lead students to the need for the formal semiotics of mathematics. (p. 3)

Support for PF methods is growing as empirical investigations amass results showing that formalisms can be learned following early, concrete experiences. Field-based research supports the claims. Romberg and Shafer (2008) used both cross-sectional and longitudinal data to document positive effects on middle school student learning with the *Mathematics in Context* curriculum. Also at the middle school level, Nathan, Stephens, and colleagues (2002) described an extended classroom intervention, using a form of PF called *Bridging Instruction*, which showed greater gains in algebra performance among seventh and eighth graders ($N = 82$) than the standard curriculum.

More carefully controlled lab studies also show support for PF. Koedinger and Anderson (1998) compared the order with which students’ problem-solving strategies were guided toward finding the unknown value for a story problem and producing a general model. One group followed the FF trajectory from formal variables and algebraic equations to specific values that were applied to the symbolic equation. In contrast, the *inductive support* method used PF by guiding students to induce the symbolic expression from prior experiences with simpler arithmetic relations. Although students in both groups showed significant gains, those gains exhibited by the inductive support group were reliably larger than for the FF group.

Goldstone and Son (2005) provided experimental evidence in favor of PF (“concreteness fading,” in their terms) for supporting transfer by comparing the order with which learners engaged with more or less formal representations of the intended mathematical structures. In this case, undergraduates were learning about the principles by which complex adaptive systems operate. Thus, the target of instruction was relatively abstract and intended to be generalizable. Transfer measures were highest for those who first learned to apply the principles in simulations using ants with very simple search rules for collectively seeking out food as concrete depictions of the systems phenomenon (a PF approach) followed by use of more abstract forms as compared to any of the other three conditions (first abstract then concrete, first abstract then abstract, and first concrete then concrete). The authors concluded that concreteness fading is effective “because it allows simulation elements to be both intuitively connected to their intended interpretations but also idealized in a manner that promotes transfer” (p. 99). In essence, PF leverages the most powerful advantages of the two representational formats: Concrete entities are meaningful to learners early on and so provide accessible entry points, abstractions transcend the applicability of the representations and rules from any one context, and *grounded abstractions* support learners’ understanding of what the formalisms “say” and how they apply widely to new application areas.

The comparative advantages of formalisms and grounded representations are apparent in an experiment comparing the types of feedback to foster algebra learning in a video-game-like environment developed by Nathan (1998) aimed at grounding the meaning of symbols and operators in algebraic equations to animations under the student's control. Grounding the algebraic equations to specific entities in student-constructed animations (such as trains colliding or overtaking one another, and the proportion of a fence that was painted working alone or with a coworker) helped students to learn more efficiently and more effectively about the meaning of those quantitative relations that were perceptually salient to students. Students who received no explicit grounding of the equations to the animated situation of typical algebra word problems made more conceptual errors and showed lower test gains and less transfer overall than students who received grounding. However, some conceptual aspects of the quantitative structure that could not be presented concretely to learners (in one case, the rather tacit algorithm of computing the reciprocal of the sum of reciprocals of related rates) saw no benefit from grounding the equations to the animation of the problem situation. Direct instruction in the formal symbol manipulation process, through a set of pre-scripted hints, showed lower learning gains overall but was significantly more effective than the approach that used animation-based grounding in reducing the frequency of errors that were not perceptually salient in the animations. This illustrates some of the relative benefits of grounding and formalism-based instruction and the trade-offs for fostering conceptual development.

Project-based and problem-based learning (PBL) and inquiry learning also offer alternative approaches that promote student-centered environments oriented toward intellectual engagement in authentic scientific and mathematical thinking and the production of authentic artifacts, including use and derivation of formalisms (e.g., Dochy, Segers, Van den Bossche, & Gijbels, 2003; Marx, Blumenfeld, Krajcik, & Soloway, 1997). When PBL and inquiry learning are taught in a way that guides students' learning and employs effective scaffolding practices (Hmelo-Silver, Duncan, & Chinn, 2007), students show benefits in both skills and content knowledge (Roth & Roychoudhury, 1993). Of particular relevance is the sequencing of learner activities and its impact on student performance. For instance, in medical education, where PBL has its roots, PBL students tend to be exposed to less formal science content (e.g., biochemistry, anatomy), and their exposure to this content occurs later in their studies than students in conventional programs, because of PBL's early commitment to more applied aspects of medical training (e.g., palpation, collecting blood pressure measurements). Yet a recent meta-analysis of more than 270 comparisons for a single, long-standing PBL-based medical program showed that, though PBL students initially exhibit lower levels of formal science knowledge than randomly selected students from the conventional program, PBL students' science knowledge continued to increase after course taking, whereas PBL students also exhibited more accurate diagnostic reasoning, superior integration of biomedical knowledge with clinical practices, higher levels of communication skills, and better domain-specific practical medical skills, such as performing examinations (Schmidt, Van Der Molen, Te Winkel, & Wijnen, 2009). From this perspective, PBL provides an insightful counternarrative to the FF account of conceptual development that posits the necessary role of early formal instruction for engendering successful applied, clinical practice.

Methods for grounding the learning of formal representations such as PBL, inquiry learning, and PF offer viable alternatives to FF approaches for designing curricula and learning experiences for novices. They support deeper understanding and more lasting learning by building on the strengths that grounded and formal representations each contribute (H. T. Tabachneck et al., 1995). Grounded representations and strategies will generally be computationally less efficient than formal methods, but their direct mapping to the referent situation promotes meaning making and facilitates learner-guided error detection and correction (Nathan, 1998). In a complementary way, the standardized syntax of formal representations increases computational efficiency. Also, the larger conceptual distance of formal representations from the referent situation enhances its universal applicability, thereby fostering generalization and abstraction.

Approaches such as PF, inquiry learning, and problem-based/project-based learning strive to bring complementary aspects of concrete entities and experiences and abstract rules and representations together. When integrated, these approaches acknowledge the important role that applied performance and concrete entities play in providing novices accessible entry points that support early comprehension and meaning making, and then exploit the generalizability and computational efficiencies of formalisms and, synergistically, foster an understanding of the formalisms. As research matures, we will likely learn more about the nature of student learning and conceptual development and uncover the processes that mediate integrative representations, such as those proposed by Case and colleagues (Case, 1991; Case & Okamoto, 2000; Griffin, Case, & Siegler, 1994; Kalchman & Case, 1998; Kalchman, Moss, & Case, 2001; Siegler & Ramani, 2009).

DISCUSSION

In this article, I delved into a belief that appears to be ubiquitous in formal education and highly influential for shaping areas such as curriculum design, classroom instruction, and expectations of learning yet has received little scrutiny in education research. At the heart of this view is the special regard afforded formal representations and belief in the primacy of formalisms in conceptual development and instruction. In this final section I want to explore the issue more broadly, entertain some speculative claims about FF, and discuss implications of the preceding analysis for instruction and the general aims of public education.

In challenging the broad application and influence of the FF view, I find it important to also acknowledge the ways in which it is effective. These are the limited circumstances that favor experts operating in domains in which they exhibit mastery. Experts are remarkable agents and seem to be able to perform extraordinary intellectual and physical feats (Ericsson & Smith, 1991). It is natural to hold up expert performance as the target for education (Glaser, 1990). However, we must carefully distinguish the needs of novices with those of experts. In the hands of experts, and even those who are "merely" highly competent in their field, formalisms are concise, tractable, and highly efficient ways of achieving one's goals and expressing knowledge. Some might even say that formalisms (ironically) can *ground* ideas for experts by facilitating ties between new ideas to first principles.[3]

When used properly by masterful practitioners, formalisms are practically invisible to the agent. It is only when formalisms are used by novice learners do we see just how laden these representations are with arbitrary attributions and domain-specific knowledge. (For illustrations of how formalisms can be misapplied because of poor understanding of their underlying meaning, see Koedinger & Nathan, 2004; Ma, 1999; and VanLehn, 1990.) In Heidegger's terminology (Dreyfus, 1991), a formal representation is *ready-to-hand* to an expert, who wields it like a skilled carpenter wields a hammer; and it is so integral to the execution of the task, and the process is so well-practiced, that the expert sees through the tool to the task itself. Though hammers are certainly not formalisms, one can appreciate, by analogy, the interruption and shift in attention to the hammer that is suddenly *unready-at-hand*, such as when the hammer breaks or a nail malfunctions (cf. Dotov, Nie, & Chemero, 2010). Formalisms in the hands of novices often have this *unready-at-hand* quality where they easily "break" and suddenly appear to be cryptic or ill-suited to the task. For example, consider how high school students ($N1 = 76$, $N2 = 171$), all with at least 1 year of algebra education, gave no response more than 30% of the time when problems were presented as symbolic equations—more than twice the rate of other presentation formats (Koedinger & Nathan, 2004). In such cases formalisms resist becoming transparent and can even be a hindrance to the task goals.

Although no systematic studies of general teacher education programs have been conducted with this analytic perspective, I speculate that the FF view influences curriculum design even in my own field of educational psychology. As a preliminary examination of this claim, I reviewed all of the educational psychology textbooks from my campus curriculum library published in the last 15 years that were designed for teacher education courses ($N = 8$). I looked specifically at the sequences of topics across each textbook, the location where the first substantive treatment of actual teaching practices appears, and the sequences within each chapter of theory to application for a given topic. To be considered "substantive," treatment of instructional practices had to exceed one page and had to address a specific grade level and one or more specific content areas. I also applied the broad view of formalisms, which included formal psychological theories, computational models, and idealized psychological constructs, in addition to symbolic representations.

With these assumptions in place, the educational psychology textbooks show many traits of FF. Many teaching practices are discussed throughout each book, but they are presented in the abstract with respect to age or content (e.g., a caution not to exceed short-term memory) or far too briefly (a sentence or two) to provide sufficient detail to be adopted by novices. On average, the first substantive treatment of applied teaching practices appears 66.8% of the way into each book (ranging from 46.5% to 81%). Analysis of the sequences of topics shows this arises because the order of major topics proceeds similarly across most of the textbooks: a broad introduction, followed by theories (and theorists) in development; theories of cognition, motivation, and learning; classroom instruction and class management (where the substantive treatment of instruction typically appears); assessment; and diversity among learners. Looking within chapters, it was common to conclude with brief, nonsubstantive treatments of instructional implications, though three books in the sample also interleaved or started with specific, motivating cases of learning, development, or instruction. Although these findings are only provisional, the patterns stand in contrast to the focus on hands-on, situated teaching knowledge that dominates curriculum and instruction methods courses. Indeed, this point is also made in a recent report from the American Psychological Association Division 15 (Educational Psychology) charged with revisiting the American Psychological Association recommendations from the 1995 committee charged with examining the role of educational psychology in teacher education (Patrick, Anderman, Bruening, & Duffin, 2011). In their report, Patrick and colleagues noted that research in educational psychology is too often largely abstracted away from the actual practices and issues facing teachers, opting instead for generalized theories and principles of behavior. In addressing the challenge of *relevance* of educational psychology to teaching, they cautioned that "educational psychologists cannot assume that simply deriving a list of principles from relatively decontextualized studies is appropriate for making recommendations to practitioners" (p. 75).

There is no question that the broad focus on formal psychological theories, constructs, and principles is quite different from versions of FF discussed up to this point, such as the symbol precedence of algebra, which are based on the narrow view of formalisms. Theories and principles are generally couched in linguistic rather than symbolic or diagrammatic forms. However, I propose that the general expectation is quite similar: Knowledge of decontextualized concepts of a scientific domain (in this case, psychology) presented in

[3]This exact point was made by an anonymous reviewer, whom I wish to acknowledge.

formal language (specialized terms and diagrams) must be mastered prior to developing competencies in the relevant applied area (teaching). From this perspective, it would seem radical to imagine imparting professional teaching practices prior to establishing a foundation in formal psychological theories. Teaching, in this light, is seen as the classroom application of psychological theories of development, learning and motivation, instructional approaches, and assessment.

Recently this "foundations first" model of educational psychology within teacher education programs has come under criticism. Too frequently, learning the formal theories and principles of human behavior out of context from instruction and other classroom-based practices has led to weak application of these ideas to the real problems that are encountered by classroom teachers (Anderson et al., 1995; Berliner, 1992; Nathan & Alibali, 2010). However, as a developmental structure, elements of FF still appear to be prevalent in the designs of many widely adopted educational psychology textbooks and their associated programs. Along these lines, I offer similar remedies in educational psychology to those in other content areas. Teacher education textbooks are drawing more frequently on the use of case books and video libraries that situate the behaviors referred to by psychological theories and ground their meaning and relevance. This practice should be encouraged. Caution must also be exercised, however, that these resources are used integrally with teaching psychological theory. There is danger, evident in Sherman's (2010) work in mathematics education, that a disconnection between the intended curriculum of the textbook is not reflected in the enacted curriculum in the teacher education classroom, and, despite these innovations, a foundations first approach may still dominate the teacher education experience. Ultimately, a PF approach to educational psychology instruction would lead to rebuilding these textbooks from the ground up.

If FF is problematic, then why does it persist? Certainly one powerful influence is the deeply entrenched societal view, inherited from early Western philosophical views, that privileges abstract knowledge and work with symbols. I suspect that another major reason is the value it serves content area experts. It is often content experts who forge the broad outlines of curriculum design and instructional approaches, and their views are weighed heavily when reviewing such designs. Experts are steeped on discipline-based perspectives, which they contribute to and internalize. These views, sometimes to the exclusion of developmental and pedagogical considerations, shape the externalization of these structures in the form of curriculum and instruction. Kline (1973) made a similar observation in deconstructing the basis of the New Math movement and its emphasis on Modern Mathematics and rigorous proof as a pathway to students' conceptual development. In promoting these formalism-based approaches, in his view, members of the mathematics community "are serving themselves" (p. 94). He argued, "It is rational to present mathematics logically, but it is not wise. . . . The wise man would also consider whether young people can learn the theory" (p. 89). This suggests that to change teachers' views on this matter, we may need to do more than provide them with reform curricula that advocate alternative approaches to FF. Teachers' views will likely continue to reflect those perspectives that are pervasive in the media and in beliefs and values espoused in society at large.

Implications for Instruction

In many ways, seemingly opposing approaches to instruction such as FF and PF ultimately strive for a common goal, whereby learners develop a deep and flexible understanding of a powerful set of representations that can propel their thinking and reveal underlying structural similarities across a broad range of disciplines and activities. Despite the challenges students encounter in understanding and learning to use formalisms, both narrowly and broadly construed (diSessa, 2004), the utility of these specialized representations for mastering a field is unmatched. Where FF and PF differ most dramatically is in conceptualizing the *paths* along which learner development proceeds.

In making prescriptive claims, FF identifies early introduction to formalisms as the most effective entry point for learning a new domain. In this way, the formalisms, devoid of distracting features or associations with particular contexts, depict the pure structure of relations as the curricular target. The formalism under the FF view is not only "fronted" as a gatekeeper into a field, but it is "centered" as well, serving as the focal point for instruction and later learning. Application tasks are, generally, applicable insomuch as they reinforce the understanding of the formalism and help to establish the formalism's broad utility.

As previously noted, although FF views are often found among teachers and textbooks (and even members of the research community!), there are several serious problems with the FF view that limit its promotion of learning. Perhaps the most notable to emerge from some of the studies reviewed is that FF is rarely an accurate model of learners' developmental trajectory (e.g., Nathan & Koedinger, 2000c). Learners not only stumble over early introduction to formalisms, they dislike them (Bond-Robinson, 2002) and can exhibit higher levels of performance with alternative presentation formats (Heckler, 2010; Koedinger & Nathan, 2004; Meltzer, 2005). It is really when problems become sufficiently complex that concrete formats show difficulty scaling up and formalisms show their mettle (Koedinger et al., 2008; Nathan & Kim, 2007).

Methods such as inquiry-based learning and PBL can also fall prey to FF views, when understanding of formal models of scientific phenomena are taken as primary to the learning experience. For example, analyses of classroom learning (Prevost et al., 2009) show that even teachers operating in project-based classes may still orient their curriculum units around the initial introduction of formalisms, such as formal presentations of the laws of kinematics when designing and building ballistics devices like catapults. During postlesson interviews, these teachers justify the central and early role of formalisms as necessary prerequisites to guide students' engineering designs, and the building, testing, and redesign of their devices. PF, as an alternative, promotes the idea of framing new content from the outset in an applied context that is often more likely to connect to students' prior knowledge of the setting (i.e., many high school students can build a catapult but may not understand how the angle of release and the initial velocity directly relate to the distance traveled). This, in turn, can engage students' verbal and spatial reasoning and meaning-making abilities (Abrahamson, Gutiérrez, Lee, Reinholz, & Trninic, 2011; Goldstone & Son, 2005; Koedinger & Nathan, 2004).

Connecting formal representations to applications establishes the grounding that is often missing for students who are learning the meaning and utility of formalisms (Nathan, 2008). Yet the manner of connection is also important and must be thoughtfully attended to. Too often, mere juxtaposition of formalisms and applications are used as a proxy for producing coherent integration among learners (Nathan, Alibali, Wolfgram, Srisuruchan, & Walkington, 2011). Current research suggests that explicit links need to be made by teachers or knowledgeable others to engineer learning environments with the proper level of cohesion to suit learners' needs and task goals (Alibali & Nathan, in press; McNamara, Graesser, & Lourwerse, 2012; Nathan et al., 2011; Prevost et al., 2009). Early on, cohesion of the environment must be actively produced and maintained to engender learners' formation of coherent mappings between formalisms and entities in the world. Linking moves—often established through teachers' gestures and speech acts (Alibali & Nathan, in press; Nathan, 2008)—are important means to produce a cohesive learning environment. During complex classroom projects, such as those encountered in engineering courses, students must understand the connections between formal mathematical equations and physical laws, designs and specifications, instruments and tools, objects, and device behavior. In these PBL environments, links through gesture and speech are frequently made to produce and maintain the cohesion needed to help students see the many elements and phases as contributing to an integrated whole.

Establishing cohesion provides a solid foundation for learners to realize the relations between formal and concrete entities. Over time, through methods such as concreteness fading, the emphasis shifts and favors the computational efficiencies that emerge by working directly with the formalisms. In successful cases, the uses of formalisms is mediated by a connected understanding that will enable the learner to reinvoke the grounding circumstances in order to address breakdowns, generate meaningful explanations and interpretations, or even invent customized representations that are designed to model novel situations (diSessa, 2004).

Misconceptions about the developmental process can lead teachers, curriculum designers, and educational leaders toward nonoptimal or even inaccurate conclusions about the progress of a child. FF-oriented curricula or assessment designs will base the intended sequencing on their models of learning and development (e.g., Pellegrino, Chudowsky, & Glaser, 2001). For example, a teacher or school operating with a strong FF view may use poor performance with formalisms such as symbol manipulation to withhold applications such as story problems or project-based tasks, even though the student might be able to demonstrate competency in the more applied realm. A teacher may also take a students' correct answer on a formalism-based problem as an unimpeachable indicator that the student has learned both the taught method and its broader applicability, thereby perpetuating a "veneer of accomplishment" (Lave, as cited in Hennessy, McCormick, & Murphy, 1993). PF can create learning opportunities that motivate the uses of formalisms and potentially ground them to meaningful experiences and prior knowledge.

In speculating on whether the efficacy of the FF approach might be context dependent, I consider context along three dimensions. In comparing educational settings, my expectation is that it is more prevalent in formal, *in-school*, settings than out-of-school contexts. However, this relative difference should not suggest that informal settings are absent FF. For example, the patterns found in many math, chemistry, and physics textbooks can also be found in books on craft knowledge such as kite making, where, in one instance (Hosking, 1992), the author begins with a scientific primer on lift and air pressure in the opening chapter laden with symbolic formalisms, complete with vectors, diagrams with angles of attack, and schematic depictions of airflow. Content can be considered a second dimension. FF does appear to be most pronounced in mathematics courses. But it seems prevalent throughout the STEM fields, including areas such as precollege engineering (Prevost et al., 2009). As shown, there is evidence for FF in the social sciences and in language arts, as well. The final dimension I consider is grade level. I believe that FF is most apparent in tertiary and graduate education (Laurillard, 2001), and least present in the primary grades. Further explorations of this construct across settings, curriculum materials, and staff development programs will be needed to further illuminate these assertions.

Implications for Enacting the Aims of Public Education

One concern about the formalisms first view of conceptual development is that it stands to undermine central tenets of public education. Public education is entrusted with providing equal access to excellent educational opportunities to enhance one's intellectual development and economic preparedness. But, in practice, the knowledge and skills needed to perform technical and service trades, such as carpentry, hairstyling, plumbing, and welding, among others, are generally held in lower regard within the public education system than the knowledge associated with academically oriented disciplines (e.g., mathematics, physics, biology, chemistry) that focus on the acquisition and replication of knowledge through discipline-specific formalisms (Rose, 2004). Across a wide range of areas, knowledge of formal representations is privileged over practical and applied knowledge as measured by status and pay (Symonds, Schwartz, & Ferguson, 2011). Consequently, there are biases that elevate the status of fields that primarily function around formalisms and theoretical constructs over those fields that emphasize applied knowledge and manual skills. These biases seem to be historically rooted rather than based on an empirical understanding of how students' conceptual development occurs in a given field, or the relative level of intellectual demands incurred by jobs across pay grades and organizational levels. Judgments of the relative worth of fields of study and occupations seem to be based largely on the perceptions of intellectual and practical demands. Rose (2004) described this perception in stark terms: "The more applied and materialized the mathematics is, the less intellectually substantial it is" (p. 98).

In actuality, analyses of nonprofessional trade skills and knowledge reveal task performance rich in cognitive complexities, such as planning, representation use, analytical thinking, and reflection (Beach, 1993; Rose, 2004; Scribner, 1984). Yet the split between the practical work of the "hand" and the intellectual work of the "head" is woven into the very fabric of people's thinking about science in the modern age.

Not long ago, vocational education programs focused on student learning of the applied skills in order that young people could secure certain roles in the workplace. In practice, however, "voc ed" courses contributed to the polarization between college- and career-ready graduates of secondary education. Rose (2004) called this the "fundamental paradox of vocational education," and argued that the persistent lack of attention to theory, generalization, and reflection in current technical education classes withholds essential knowledge while perpetuating stereotypes of who is capable of abstract thought and worthy of the tremendous resources of the educational system to foster upward economic and social mobility. Recent shifts in policy and legislation are intended to recast traditional vocational education as Career and Technical Education. The 2006 Reauthorization of the Perkins Career and Technical Education Act (Public Law 105–332, 1998; previously the Perkins Vocational and Technical Education Act) mandates that technical education and academic subjects (e.g., math, physics) must be integrated "so that students achieve both academic and occupational competencies" with substantial funds allocated "to provide vocational education programs that integrate academic and vocational education."

As we have seen, the way this integration happens appears to be driven, in part, by tacit beliefs about learning that shape instruction and curriculum design. This has consequences for the learning experiences students obtain in these programs. Furthermore, in keeping with the elevated status of those second-order experiences that dominate academic discourse, the FF view functions to distance those with scholarly and professional aspirations from the first-order experiences that often serve as the grounding experiences needed for deeply understanding and applying those formal representations. Because they are abstractions and not contextually bound, formal representations are better suited for generalizing the underlying ideas to a broad range of contexts (Son & Goldstone, 2009). Yet, for profound understanding, both forms of knowing must ultimately be achieved (Laurillard, 2001; Ma, 1999; L. B. Resnick, 1987). Beliefs in the primacy of formalisms, and the concomitant policies and curricular designs that follow from these beliefs, contribute to the segregation of these forms of learning. In so doing, they undermine the egalitarian aims of public education by keeping some learners away from the very reality that formalisms are intended to describe and keeping others from developing a grounded understanding of the types of generalized thinking that are most highly rewarded.

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